Gravitation

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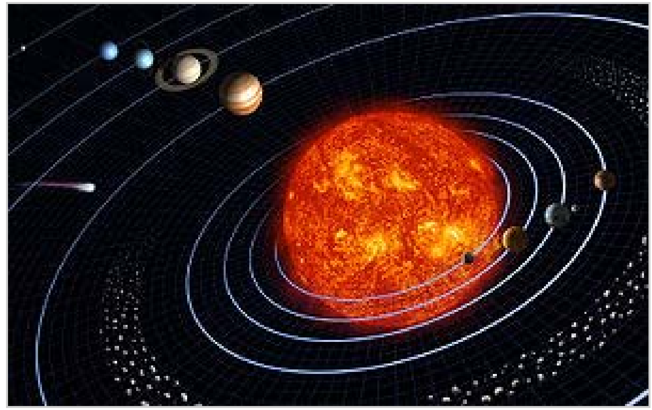
(Redirected from Gravity)



Gravitation keeps the planets in orbit around the Sun. (Not to scale)

Gravitation, or **gravity**, is a natural phenomenon in which objects with mass attract one another.[1] In everyday life, gravitation is most familiar as the agent that gives weight to objects with mass and causes them to fall to the ground when dropped. Gravitation causes dispersed matter to coalesce, thus accounting for the existence of the Earth, the Sun, and most of the macroscopic objects in the universe. Gravitation is responsible for keeping the Earth and the other planets in their orbits around the Sun; for keeping the Moon in its orbit around the Earth; for the formation of tides; for natural convection, by which fluid flow occurs under the influence of a density gradient and gravity; for heating the interiors of forming stars and planets to very high temperatures; and for various other phenomena observed on Earth.

Gravitation is one of the four fundamental interactions of nature, along with the strong force, electromagnetism and the weak force. Modern physics describes gravitation using the general theory of relativity, in which gravitation is a consequence of the curvature of spacetime which governs the motion of inertial objects. The simpler Newton's law of universal gravitation provides an accurate approximation for most calculations.

Contents

History of gravitational theory

Main article: History of gravitational theory

Scientific revolution

Modern work on gravitational theory began with the work of Galileo Galilei in the late 16th and early 17th centuries. In his famous (though possibly apocryphal[2]) experiment dropping balls from the Tower of Pisa, and later with careful measurements of balls rolling down inclines, Galileo showed that gravitation accelerates all objects at the same rate. This was a major departure from Aristotle's belief that heavier objects accelerate faster.[3] Galileo correctly postulated air resistance as the reason that lighter objects may fall more slowly in an atmosphere. Galileo's work set the stage for the formulation of Newton's theory of gravity.

Newton's theory of gravitation

Main article: Newton's law of universal gravitation

In 1687, English mathematician Sir Isaac Newton published *Principia*, which hypothesizes the inverse-square law of universal gravitation. In his own words, “I deduced that the forces which keep the planets in their orbs must [be] reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the Moon in her Orb with the force of gravity at the surface of the Earth; and found them answer pretty nearly.”[4]

Newton's theory enjoyed its greatest success when it was used to predict the existence of Neptune based on motions of Uranus that could not be accounted for by the actions of the other planets. Calculations by both John Couch Adams and Urbain Le Verrier predicted the general position of the planet, and Le Verrier's calculations are what led Johann Gottfried Galle to the discovery of Neptune.

A discrepancy in Mercury's orbit pointed out flaws in Newton's theory. By the end of the 19th century, it was known that its orbit showed slight perturbations that could not be accounted for entirely under Newton's theory, but all searches for another perturbing body (such as a planet orbiting the Sun even closer than Mercury) had been fruitless. The issue was resolved in 1915 by Albert Einstein's new theory of general relativity, which accounted for the small discrepancy in Mercury's orbit.

Although Newton's theory has been superseded, most modern non-relativistic gravitational calculations are still made using Newton's theory because it is a much simpler theory to work with than general relativity, and gives sufficiently accurate results for most applications involving sufficiently small masses, speeds and energies.

Equivalence principle

The equivalence principle, explored by a succession of researchers including Galileo, Loránd Eötvös, and Einstein, expresses the idea that all objects fall in the same way. The simplest way to test the weak equivalence principle is to drop two objects of different masses or compositions in a vacuum, and see if they hit the ground at the same time. These experiments demonstrate that all objects fall at the same rate when friction (including air resistance) is negligible. More sophisticated tests use a torsion balance of a type invented by Eötvös. Satellite experiments are planned for more accurate experiments in space.[5]

Formulations of the equivalence principle include:

- The weak equivalence principle: *The trajectory of a point mass in a gravitational field depends only on its initial position and velocity, and is independent of its composition.*[6]
- The Einsteinian equivalence principle: *The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in spacetime.*[7]
- The strong equivalence principle requiring both of the above.

The equivalence principle can be used to make physical deductions about the gravitational constant, the geometrical nature of gravity, the possibility of a fifth force, and the validity of concepts such as general relativity and Brans-Dicke theory.

General relativity

See also: Introduction to general relativity

In **general relativity**, the effects of gravitation are ascribed to spacetime curvature instead of a force. The starting point for general relativity is the equivalence principle, which equates free fall with inertial motion, and describes free-falling inertial objects as being accelerated relative to non-inertial observers on the ground.[8][9] In Newtonian physics, however, no such acceleration can occur unless at least one of the objects is being operated on by a force.

Einstein proposed that spacetime is curved by matter, and that free-falling objects are moving along locally straight paths in curved spacetime. These straight paths are called geodesics. Like Newton's first law of motion, Einstein's theory states that if a force is applied on an object, it would deviate from a geodesic.[10] For instance, we are no longer following geodesics while standing because the mechanical resistance of the Earth exerts an upward force on us, and we are non-inertial on the ground as a result. This explains why moving along the geodesics in spacetime is considered inertial.

Einstein discovered the field equations of general relativity, which relate the presence of matter and the curvature of spacetime and are named after him. The Einstein field equations are a set of 10 simultaneous, non-linear, differential equations. The solutions of the field equations are the components of the metric tensor of spacetime. A metric tensor describes a geometry of spacetime. The geodesic paths for a spacetime are calculated from the metric tensor.

Notable solutions of the Einstein field equations include:

- The Schwarzschild solution, which describes spacetime surrounding a spherically symmetric non-rotating uncharged massive object. For

General relativity

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Einstein field equations

Introduction
Mathematical formulation
Resources

Fundamental concepts

Special relativity
Equivalence principle
World line · Riemannian geometry

Phenomena

Kepler problem · Lenses · Waves
Frame-dragging · Geodetic effect
Event horizon · Singularity
Black hole

Equations

Linearized Gravity
Post-Newtonian formalism

compact enough objects, this solution generated a black hole with a central singularity. For radial distances from the center which are much greater than the Schwarzschild radius, the accelerations predicted by the Schwarzschild solution are practically identical to those predicted by Newton's theory of gravity.
- The Reissner-Nordström solution, in which the central object has an electrical charge. For charges with a geometrized length which are less than the geometrized length of the mass of the object, this solution produces black holes with two event horizons.
- The Kerr solution for rotating massive objects. This solution also produces black holes with multiple event horizons.
- The Kerr-Newman solution for charged, rotating massive objects. This solution also produces black holes with multiple event horizons.
- The cosmological Friedmann-Lemaitre-Robertson-Walker solution, which predicts the expansion of the universe.

The tests of general relativity included the following:[11]

- General relativity accounts for the anomalous perihelion precession of Mercury.[2]
- The prediction that time runs slower at lower potentials has been confirmed by the Pound–Rebka experiment, the Hafele–Keating experiment, and the GPS.
- The prediction of the deflection of light was first confirmed by Arthur Stanley Eddington from his observations during the Solar eclipse of May 29, 1919.[12][13] Eddington measured starlight deflections twice those predicted by Newtonian corpuscular theory, in accordance with the predictions of general relativity. However his interpretation of the results was later disputed.[14] More recent tests using radio interferometric measurements of quasars passing behind the Sun have more accurately and consistently confirmed the deflection of light to the degree predicted by general relativity.[15] See also gravitational lens.
- The time delay of light passing close to a massive object was first identified by Irwin I. Shapiro in 1964 in interplanetary spacecraft signals.
- Gravitational radiation has been indirectly confirmed through studies of binary pulsars.
- Alexander Friedmann in 1922 found that Einstein equations have non-stationary solutions (even in the presence of the cosmological constant). In 1927 Georges Lemaître showed that static solutions of the Einstein equations, which are possible in the presence of the cosmological constant, are unstable, and therefore the static universe envisioned by Einstein could not exist. Later, in 1931, Einstein himself agreed with the results of Friedmann and Lemaître. Thus general relativity predicted that the Universe had to be non-static—it had to either expand or contract. The expansion of the universe discovered by Edwin Hubble in 1929 confirmed this prediction.[16]

Einstein field equations
Friedmann equations
ADM formalism
BSSN formalism

Advanced theories

Kaluza–Klein
Quantum gravity

Solutions

Schwarzschild
Reissner-Nordström · Gödel
Kerr · Kerr-Newman
Kasner · Taub-NUT · Milne · Robertson-Walker
pp-wave

Scientists

Einstein · Minkowski · Eddington
Lemaître · Schwarzschild
Robertson · Kerr · Friedman
Chandrasekhar · Hawking
· *others*

Gravity and quantum mechanics

Main articles: Graviton and Quantum gravity

In the decades after the discovery of general relativity it was realized that general relativity is incompatible with quantum mechanics.[17] It is possible to describe gravity in the framework of quantum field theory like the other fundamental forces, such that the attractive force of gravity arises due to exchange of virtual gravitons, in the same way as the electromagnetic force arises from exchange of virtual photons.[18][19] This reproduces general relativity in the classical limit. However, this approach fails at short distances of the order of the Planck length,[17] where a more complete theory of quantum gravity (or a new approach to quantum mechanics) is required. Many believe the complete theory to be string theory,[20] or more currently M-theory, and, on the other hand, it may be a background independent theory such as loop quantum gravity or causal dynamical triangulation.

Specifics

Earth's gravity

Main article: Earth's gravity

Every planetary body (including the Earth) is surrounded by its own gravitational field, which exerts an attractive force on all objects. Assuming a spherically symmetrical planet (a reasonable approximation), the strength of this field at any given point is proportional to the planetary body's mass and inversely proportional to the square of the distance from the center of the body.

The strength of the gravitational field is numerically equal to the acceleration of objects under its influence, and its value at the Earth's surface, denoted g, is approximately expressed below as the standard average.

$g = 9.81\ \mathrm{m/s^2} = 32.2\ \mathrm{ft/s^2}$

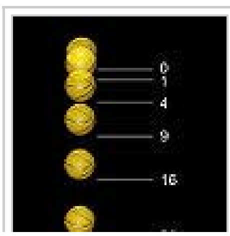
This means that, ignoring air resistance, an object falling freely near the Earth's surface increases its velocity by 9.81 m/s (32.2 ft/s or 22 mph) for each second of its descent. Thus, an object starting from rest will attain a velocity of 9.81 m/s (32.2 ft/s) after one second, 19.6 m/s (64.4 ft/s) after two seconds, and so on, adding 9.81 m/s (32.2 ft/s) to each resulting velocity. Also, again ignoring air resistance, any and all objects, when dropped from the same height, will hit the ground at the same time.

According to Newton's 3rd Law, the Earth itself experiences an equal [in force] and opposite [in direction] force to that acting on the falling object, meaning that the Earth also accelerates towards the object (until the object hits the earth, then the Law of Conservation of Energy states that it will move back with the same acceleration with which it initially moved forward, canceling out the two forces of gravity.). However, because the mass of the Earth is huge, the acceleration of the Earth by this same force is negligible, when measured relative to the system's center of mass.

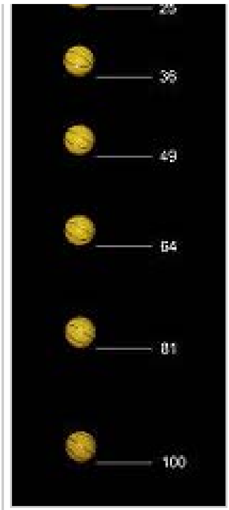
When an object with commensurable mass would "fall" then the acceleration of Earth would really be observable.

Equations for a falling body near the surface of the Earth

Main article: Equations for a falling body



Under an assumption of constant gravity, Newton's law of universal gravitation simplifies to $F = mg$, where m is the mass of the body and g is a constant vector with an average magnitude of $9.81\ \mathrm{m/s^2}$. The acceleration due to gravity is equal to this g. An initially stationary object which is allowed to fall freely under gravity drops a distance which is proportional to the square of the elapsed time. The image on the right, spanning half a second, was captured with a stroboscopic flash at 20 flashes per second. During the first $^{1}/_{20}$ of a second the ball drops one unit of distance (here, a unit is about 12 mm); by $^{2}/_{20}$ it has dropped at total of 4 units; by $^{3}/_{20}$, 9 units and so on.



Ball falling freely under gravity. See text for description.

Under the same constant gravity assumptions, the potential energy, E_p, of a body at height h is given by $E_p = mgh$ (or $E_p = Wh$, with W meaning weight). This expression is valid only over small distances h from the surface of the Earth. Similarly, the expression $h = \frac{v^2}{2g}$ for the maximum height reached by a vertically projected body with velocity v is useful for small heights and small initial velocities only.

Gravity and astronomy

Main article: Gravitation (astronomy)

The discovery and application of Newton's law of gravity accounts for the detailed information we have about the planets in our solar system, the mass of the Sun, the distance to stars, quasars and even the theory of dark matter. Although we have not traveled to all the planets nor to the Sun, we know their masses. These masses are obtained by applying the laws of gravity to the measured characteristics of the orbit. In space an object maintains its orbit because of the force of gravity acting upon it. Planets orbit stars, stars orbit Galactic Centers, galaxies orbit a center of mass in clusters, and clusters orbit in superclusters. The force of gravity is proportional to the mass of an object and inversely proportional to the square of the distance between the objects.

Gravitational radiation

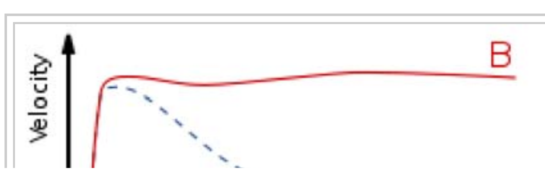
Main article: Gravitational wave

In general relativity, gravitational radiation is generated in situations where the curvature of spacetime is oscillating, such as is the case with co-orbiting objects. The gravitational radiation emitted by the Solar System is far too small to measure. However, gravitational radiation has been indirectly observed as an energy loss over time in binary pulsar systems such as PSR B1913+16. It is believed that neutron star mergers and black hole formation may create detectable amounts of gravitational radiation. Gravitational radiation observatories such as LIGO have been created to study the problem. No confirmed detections have been made of this hypothetical radiation, but as the science behind LIGO is refined and as the instruments themselves are endowed with greater sensitivity over the next decade, this may change.

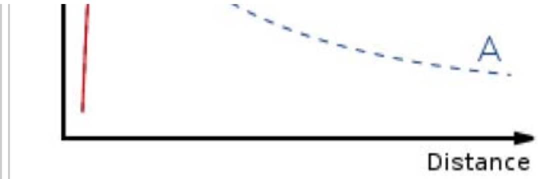
Anomalies and discrepancies

There are some observations that are not adequately accounted for, which may point to the need for better theories of gravity or perhaps be explained in other ways.

- **Extra fast stars**: Stars in galaxies follow a distribution of velocities where stars on the outskirts are moving faster than they should according to the observed distributions of normal matter. Galaxies within galaxy clusters show a similar pattern.



Dark matter, which would interact gravitationally but not electromagnetically, would account for the discrepancy. Various modifications to Newtonian dynamics have also been proposed.



Rotation curve of a typical spiral galaxy: predicted (**A**) and observed (**B**). The discrepancy between the curves is attributed to dark matter.

- **Pioneer anomaly**: The two Pioneer spacecraft seem to be slowing down in a way which has yet to be explained.[21]
- **Flyby anomaly**: Various spacecraft have experienced greater accelerations during slingshot maneuvers than expected.
- **Accelerating expansion**: The metric expansion of space seems to be speeding up. Dark energy has been proposed to explain this. A recent alternative explanation is that the geometry of space is not homogeneous (due to clusters of galaxies) and that when the data are reinterpreted to take this into account, the expansion is not speeding up after all,[22] however this conclusion is disputed.[23]
- **Anomalous increase of the astronomical unit**: Recent measurements indicate that planetary orbits are widening faster than if this was solely through the sun losing mass by radiating energy.
- **Extra energetic photons**: Photons travelling through galaxy clusters should gain energy and then lose it again on the way out. The accelerating expansion of the universe should stop the photons returning all the energy, but even taking this into account photons from the cosmic microwave background radiation gain twice as much energy as expected. This may indicate that gravity falls off *faster* than inverse-squared at certain distance scales.[24]
- **Dark flow**: Surveys of galaxy motions have detected a mystery dark flow towards an unseen mass. Such a large mass is too large to have accumulated since the Big Bang using current models and may indicate that gravity falls off *slower* than inverse-squared at certain distance scales.[24]
- **Extra massive hydrogen clouds**: The spectral lines of the Lyman-alpha forest suggest that hydrogen clouds are more clumped together at certain scales than expected and, like dark flow, may indicate that gravity falls off *slower* than inverse-squared at certain distance scales.[24]

Alternative theories

Main article: Alternatives to general relativity

Historical alternative theories

- Aristotelian theory of gravity
- Le Sage's theory of gravitation (1784) also called **LeSage gravity**, proposed by Georges-Louis Le Sage, based on a fluid-based explanation where a light gas fills the entire universe.
- Nordström's theory of gravitation (1912, 1913), an early competitor of general relativity.
- Whitehead's theory of gravitation (1922), another early competitor of general relativity.

Recent alternative theories

- Brans–Dicke theory of gravity (1961)
- Induced gravity (1967), a proposal by Andrei Sakharov according to which general relativity might arise from quantum field theories of matter
- In the modified Newtonian dynamics (MOND) (1981), Mordehai Milgrom proposes a modification of Newton's Second Law of motion for small accelerations
- The self-creation cosmology theory of gravity (1982) by G.A. Barber in which the Brans-Dicke theory is modified to allow mass creation
- Nonsymmetric gravitational theory (NGT) (1994) by John Moffat
- Tensor-vector-scalar gravity (TeVeS) (2004), a relativistic modification of MOND by Jacob Bekenstein
- Gravity as an entropic force, gravity arising as an emergent phenomenon from the thermodynamic concept of entropy.

See also

- Anti-gravity, the idea of neutralizing or repelling gravity
- Artificial gravity
- Escape velocity, the minimum velocity needed to fly away from a massive space object
- g-force, a measure of acceleration
- Gravitational induction
- Gravitational binding energy
- Gravity assist
- Gravity Recovery and Climate Experiment
- Gravity Research Foundation
- Gauss' law for gravity
- Jovian-Plutonian gravitational effect
- Kepler's third law of planetary motion
- Lagrangian point
- Mixmaster dynamics
- Newton's laws of motion
- *n*-body problem
- Pioneer anomaly
- Scalar theories of gravitation
- Speed of gravity
- Standard gravitational parameter
- Standard gravity
- Weightlessness

Notes

- ^ Proposition 75, Theorem 35: p. 956 - I.Bernard Cohen and Anne Whitman, translators: Isaac Newton, *The Principia*: Mathematical Principles of Natural Philosophy. Preceded by *A Guide to Newton's Principia*, by I. Bernard Cohen. University of California Press 1999 ISBN 0-520-08816-6 ISBN 0-520-08817-4
- ^ Max Born (1924), *Einstein's Theory of Relativity* (The 1962 Dover edition, page 348 lists a table documenting the observed and calculated values for the precession of the perihelion of Mercury, Venus, and

Earth.)

Footnotes

1. ^ Does Gravity Travel at the Speed of Light? (http://math.ucr.edu/home/baez/physics/Relativity/GR/grav_speed.html) , *UCR Mathematics*. 1998. Retrieved 3 July 2008.
2. ^ Ball, Phil (06 2005). "Tall Tales". *Nature News*. doi:10.1038/news050613-10 (http://dx.doi.org/10.1038%2Fnews050613-10) .
3. ^ Galileo (1638), *Two New Sciences*, First Day (http://oll.libertyfund.org/?option=com_staticxt&staticfile=show.php%3Ftitle=753&chapter=109891&layout=html&Itemid=27) Salviati speaks: "If this were what Aristotle meant you would burden him with another error which would amount to a falsehood; because, since there is no such sheer height available on earth, it is clear that Aristotle could not have made the experiment; yet he wishes to give us the impression of his having performed it when he speaks of such an effect as one which we see."
4. ^ *Chandrasekhar, Subrahmanyan (2003). *Newton's Principia for the common reader*. Oxford: Oxford University Press. (pp.1–2). The quotation comes from a memorandum thought to have been written about 1714. As early as 1645 Ismaël Bullialdus had argued that any force exerted by the Sun on distant objects would have to follow an inverse-square law. However, he also dismissed the idea that any such force did exist. See, for example, Linton, Christopher M. (2004). *From Eudoxus to Einstein—A History of Mathematical Astronomy*. Cambridge: Cambridge University Press. p. 225. ISBN 978-0-521-82750-8.
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9. ^ http://laser.phys.ualberta.ca/~egerton/genrel.htm
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11. ^ Pauli, Wolfgang Ernst (1958). "Part IV. General Theory of Relativity". *Theory of Relativity*. Courier Dover Publications. ISBN 9780486641522.
12. ^ Dyson, F.W.; Eddington, A.S.; Davidson, C.R. (1920). "A Determination of the Deflection of Light by the Sun's Gravitational Field, from Observations Made at the Total Eclipse of May 29, 1919" (http://adsabs.harvard.edu/abs/1920RSPTA.220..291D) . *Phil. Trans. Roy. Soc. A* **220**: 291–333. doi:10.1098/rsta.1920.0009 (http://dx.doi.org/10.1098%2Frsta.1920.0009) . http://adsabs.harvard.edu/abs/1920RSPTA.220..291D.. Quote, p. 332: "Thus the results of the expeditions to Sobral and Principe can leave little doubt that a deflection of light takes place in the neighbourhood of the sun and that it is of the amount demanded by Einstein's generalised theory of relativity, as attributable to the sun's gravitational field."
13. ^ Weinberg, Steven (1972). *Gravitation and cosmology*. John Wiley & Sons.. Quote, p. 192: "About a dozen stars in all were studied, and yielded values 1.98 ± 0.11" and 1.61 ± 0.31", in substantial agreement with Einstein's prediction θ□ = 1.75"."
14. ^ Earman, John; Glymour, Clark (1980). "Relativity and Eclipses: The British eclipse expeditions of 1919 and their predecessors". *Historical Studies in the Physical Sciences* **11**: 49–85.
15. ^ Weinberg, Steven (1972). *Gravitation and cosmology*. John Wiley & Sons. p. 194.
16. ^ See W.Pauli, 1958, pp.219–220

17. ^ *a b* Randall, Lisa (2005). *Warped Passages: Unraveling the Universe's Hidden Dimensions*. Ecco. ISBN 0-06-053108-8.
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23. ^ Swiss-cheese model of the cosmos is full of holes (http://space.newscientist.com/article/mg20026783.800-swisscheese-model-of-the-cosmos-is-full-of-holes.html) , *New Scientist*, issue 2678, 18th October 2008.
24. ^ *a b c* "Where Matter Fears to Tread", New Scientist issue 2669, 14 March 2009

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